Optimum Design of Structures with Multiple Constraints

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Automated design of large aerospace structures requires efficient optimization algorithms because of a large number of design variables and design constraints. Most of the difficulties associated with large structural design are solution convergence and computer resources requirements. Practical aerospace structures generally involve limitations on gage sizes, displacements, stresses, and frequencies. The objective of this study was to determine what techniques are reliable and efficient for optimization of a complex design problem. The study examined the relative numerical performance of various optimization methods as candidates for a hybrid algorithm using optimality criteria and mathematical programming methods. Several optimization programs were used to design truss- and wing-type structures, and the iteration history for each technique is presented. The computer programs using the method of feasible directions and a modified Newton's method of unconstrained minimization were the most reliable mathematical programming methods.

Introduction

EVEN though thousands of papers have been published on the use of optimization algorithms for engineering design, the number of applications to practical vehicles remains very small.[1] Practical structures have hundreds of design variables, thousands of implicit design constraints, and multiple local minima. If structural optimization is to become a common tool in preliminary design of aerospace structures, computer optimization codes must handle problems of this scale reliably and efficiently. This study examines a few stand-alone optimization programs and evaluates their numerical performance in handling structural design problems with multiple (types of) constraints. Of course, not every general optimization program was tested in the study.

Recent comparative studies in structural optimization by Carpenter and Smith emphasize unconstrained minimization techniques in Ref. 2 and three constrained minimization techniques in Ref. 3, concluding that sequential linear programming is the best of the latter. Sandgren and Ragsdell[4] make similar studies. Belegundu and Arora[5,6] compare several methods for which the structural test problems were not formulated in the most amenable manner, rather they were made deliberately complex in order to rigorously test the algorithms. Consequently, results for the standard 10-bar truss problem do not even converge to within 20% of the known solution. The present study differs from these earlier ones by considering (implicit, nonlinear) stress, displacement, and frequency constraints, emphasizing wing structures, and taking full advantage of the linear structural behavior by scaling to a feasible design.

Structural optimization problems traditionally have been solved by using either the mathematical programming (MP) or the optimality criteria (OC) approach. More recently, Fleury and Sander,[7] Fleury,[8] Khot et al.[9] and Arora[10] have illustrated the uniformity of the methods. The approaches were shown to be equivalent for static (displacement or stress) constraints under the following conditions: 1) both use first-order constraint approximations, 2) the OC method solves the nonlinear system of equations for the Lagrange multipliers, and 3) the MP method uses inverse design variables. The first two conditions are not met by the OC method considered here—stress constraints are not approximated (they are replaced by a generalized stiffness constraint) and no attempt is made to solve the system of equations for the Lagrange multipliers. Therefore, each method offers certain advantages and disadvantages. Furthermore, frequency constraints have not been addressed in the OC method.

The MP methods are extremely useful in defining the design problem in proper mathematical terms. The nonlinear programming methods are most effective when the design variables are few; however, in the presence of a large number of variables these methods were very slow. Several improvements alleviated the problem: the number of independent design variables was reduced using design variable linking, the number of constraints was reduced using constraint deletion (throwaway) techniques, and the number of repeated finite-element analyses was reduced using constraint approximation concepts.[11] Linking design variables before the actual structural design begins can be a difficult task since there is no information on how the design variables behave during the design cycle. Retaining only the active constraints is a natural assumption to reduce the computational costs, although deciding how many constraints to retain is critical. Oscillations occur in the convergence if too few are kept; gradient calculations become costly as more constraints are kept.

In the OC approach, the criterion is first derived for given objective and constraint functions. A recurrence relation is written on the basis of this criterion. The strategy is to iteratively solve the optimality criterion to indirectly reach the optimum solution. Although OC have been developed for many types of constraints, not much work has been done in combining them for simultaneous, multiple-constraint op-

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timization.[12] The main difficulty with OC methods is the determination of Lagrange multipliers, especially with multiple constraints. Although the rate of convergence for OC methods is initially very fast, step-size determination becomes more difficult as the design approaches the local optimum.

Ideally, a methodology that exploits the strength of both approaches might be employed in a practical system. The object of the present research effort is to develop such a design method that can efficiently optimize large structures. This paper identifies the strengths and weaknesses of various MP methods. Eventually, a hybrid optimization method using both OC and MP methods will be developed. The current work shows that the OC algorithm is most effective at the beginning of the optimization. A reasonable approach is to carry out a few iterations of an OC method and then switch to a MP method. This paper is aimed at selecting an efficient and reliable MP method for such a hybrid optimization algorithm.

Problem Statement

The optimization task is to find a vector $\mathbf{x}$ of n design variables x_i, $i=1,2\ldots,n$, which will minimize a multivariable function $f(\mathbf{x})$ subject to behavior constraints

$$g_j(\mathbf{x}) = G_j(\mathbf{x}) - \hat{G}_j \leq 0, \quad j=1,2,\ldots,m \tag{1a}$$

and side constraints

$$x_i^l \leq x_i \leq x_i^u, \quad i=1,2,\ldots,n \tag{1b}$$

In structural applications, the design variable vector $\mathbf{x}$ controls the finite-element sizes; the behavior quantities $G_j(\mathbf{x})$ are displacements, stresses, and frequencies. $\hat{G}_j$ are limits, and $f(\mathbf{x})$ represents structural weight. The following section briefly reviews the MP and OC techniques considered in this study.

Mathematical Programming Techniques

The first part of the research effort was design of structures with multiple constraints using mathematical programming methods. These techniques employ numerical search procedures for finding the constrained minimum and have a wide selection of optimization algorithms. All algorithms are based on the following iterative equation:

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \alpha_k \mathbf{S}_k, \quad k=0,1,2,\ldots \tag{2}$$

where k is the iteration number, α_k a step size, S_k a search direction, and x_0 a starting design.

In this work, four different mathematical programming techniques were used.

1) The Method of Feasible Directions[13] is a direct method of constrained optimization. It computes a search direction that simultaneously reduces the objective and stays inside the feasible domain. The search direction S is computed by solving the following maximization problem. Maximize β so that

$$\mathbf{S}^T \nabla g_j + \theta_j \beta \leq 0, \quad j \epsilon I_A \tag{3a}$$

$$\mathbf{S}^T \nabla f + \beta \leq 0, \quad \theta_j \geq 0 \tag{3b}$$

$$\|\mathbf{S}\| = 1 \tag{3c}$$

where ∇f is the gradient of the objective function, ∇g_j the gradient of the jth constraint, θ_j the push-off (weighting) factors, I_A the active constraint set, and β a variable to be maximized. The step length in Eq. (2) is found by a one-dimensional search bounded by the constraints. The Method of Feasible Directions has been implemented with the CONMIN[14] computer program.

2) The Modified Method of Feasible Directions[13] has also been used. The alterations to the Method of Feasible Directions are essentially that the push-off factors θ_j are set equal to zero (eliminating the parameter β) and a set of dependent design variables is selected for carrying out the one-dimensional search. The remaining independent variables in the one-dimensional search are updated by Newton's method. This modified method was implemented using the Automated Design Synthesis (ADS) program.[15]

3) The third algorithm is based on a sequential unconstrained minimization technique (SUMT). The SUMT algorithm transforms the constrained problem into a sequence of unconstrained problems using a quadratic extended interior penalty function formulation. A compound function $F(x,r)$ is introduced as

$$F(\mathbf{x},r) = f(\mathbf{x}) - r\sum_{j=1}^{m} p(g_j) \tag{4}$$

where r is the penalty parameter, and the function $p(g_j)$ associated with the jth constraint is defined as

$$p(g_j) = \frac{1}{g_j}, \quad g_j \leq g_0 < 0 \tag{5a}$$

$$= \frac{1}{g_0}\left[\left(\frac{g_j}{g_0}\right)^2 - 3\left(\frac{g_j}{g_0}\right) + 3\right], \quad g_j > g_0 \tag{5b}$$

$$g_0 = c\sqrt{r} \tag{5c}$$

where g_0 is the transition parameter and c a negative constant.

The function $p(g_j)$ is thus defined as an interior penalty function in most of the feasible domain. It is defined as a quadratic exterior penalty function in a small part of the feasible domain ($g_j \geq g_0$) and in the infeasible domain. The solution of the optimization problem is obtained by minimizing the function F for a decreasing sequence of r values using Newton's method with approximate second derivatives of the penalty terms.

The design vector $\mathbf{x}$, which minimizes $F(x,r)$, is found using Eq. (2) iteratively. The search direction S_k is given as

$$\mathbf{S}_k = -\mathbf{H}_k^{-1} \nabla F_k \tag{6}$$

where H_k is the Hessian matrix of the second derivatives of F. The program NEWSUMT-A[16,17] has been used for this technique.

4) Recursive quadratic programming (RQP) methods developed by Han,[18] Powell,[19] and Pschenichny and Danilin[20] are used. They develop Newton-like methods for constrained optimization, solving a quadratic programming problem for the search direction.

$$\text{Minimize } \mathbf{S}^T \nabla f + \frac{1}{2} \mathbf{S}^T \mathbf{A}(\mathbf{x},\Lambda)\mathbf{S}$$

subject to

$$g_j(\mathbf{x}) + \mathbf{S}^T \nabla g_j(\mathbf{x}) \leq 0, \quad j=1,2,\ldots,m \tag{7}$$

where $\mathbf{A}$ is a positive definite approximation to the Hessian of the Lagrange function and Λ is the vector of Lagrange multipliers.

The methods of Han and Powell require derivatives of all constraints at each iteration. Powell uses Broyden-Fletcher-Goldfarb-Shanno quasi-Newton updates to generate $\mathbf{A}$, and this method is implemented through the program VMCON.[21] The algorithm based on Pshenichny's linearization method, using gradients of active constraints only, is implemented using the IDESIGN computer program.[22,23] The authors recommend that a hybrid method (cost function bounding algorithm combined with RQP) be used with IDESIGN; however, in all cases examined here, the results for the hybrid option (or cost function alone) were no better than the RQP method. Thus, results for the RQP option only are reported.

Approximation Concepts

In order for MP methods to approach the efficiency of OC techniques, Taylor series approximations are needed. The constraint approximations replace the basic problem statement with a sequence of explicit problems that preserve the essential features of the original design optimization problem. This approximate problem can then be optimized with appropriate move limits on the design variables to ensure that the design does not stray from the region of validity for the approximations. The Taylor series approximations constructed in terms of the inverse of the physical variables are the most popular. The motivation for use of a reciprocal design space is that, for static constraints, the approximations are exact for statically determinate structures. If the internal force distribution does not vary significantly, the approximations remain accurate.

If x_i is the ith direct design variable and $v_i \equiv 1/x_i$ is defined as the inverse variable, the approximate problem can be stated as

$$\text{Minimize } f(\mathbf{v}) = \sum_{i=1}^{n} \frac{w_i}{v_i}$$

subject to the approximate constraints

$$g_j(\mathbf{v}) = g_{0_j} + \sum_{i=1}^{n} \frac{\partial g_j}{\partial v_{0_i}} (v_i - v_{0_i}) \tag{8}$$

where w_i is a constant for each design variable. The constraints for the approximate problem are linear in the reciprocal variables and their gradients are constant: $\partial g_j/\partial v_i = \partial g_j/\partial v_{0_j}$.

The same approximate problem can be expressed in terms of the direct variables x_i using the following reciprocal approximation:

$$\text{Minimize } f(\mathbf{x}) = \sum_{i=1}^{n} w_i x_i$$

subject to the approximate constraints

$$g_j(\mathbf{x}) = g_{0_j} - \sum_{i=1}^{n} \frac{\partial g_j}{\partial x_{0_i}} x_{0i}^2 \left(\frac{1}{x_i} - \frac{1}{x_{0_i}} \right) \tag{9}$$

where the constraints are nonlinear and their gradients are no longer constant,

$$\frac{\partial g_j}{\partial x_i} = \frac{\partial g_j}{\partial x_{0_i}} \left(\frac{x_{0_i}}{x_i} \right)^2 \tag{10}$$

Theoretically, the solution to the approximate problem is the same for the reciprocal and direct formulations. The difference in practice is how the optimizer handles linear vs nonlinear constraints. Generally, linear constraints are expected to be easier to handle; however, experience using the CONMIN and ADS programs with both formulations revealed that this was not always the case—both methods reproduced the same results only when the convergence parameters were made sufficiently restrictive. The IDESIGN algorithm internally linearizes the objective function as well as all of the constraints. Use of reciprocal variables severely degraded the performance of IDESIGN, so the reported results are for direct variables, Eq. (8). NEWSUMT-A always used the direct formulation with reciprocal approximations to the constraints, Eq. (9).

Optimality Criteria Techniques

Optimality criteria methods for structural optimization involve: 1) the derivation of a set of necessary conditions that must be satisfied at the optimum design, and 2) the development of an iterative redesign procedure that drives the initial trial design toward a design that satisfies the previously established set of necessary conditions.

First, the Lagrangian formulation for constrained minimization is written as

$$L(\mathbf{x}) = f(\mathbf{x}) + \sum_{j=1}^{m} \lambda_j g_j(\mathbf{x}) \tag{11}$$

where $L(x)$ is the Lagrangian function and λ the Lagrangian multiplier.

Minimization of the Lagrangian L with respect to the design variable vector x gives the condition for the stationary value of the objective function with the constraint condition g as

$$\frac{\partial L}{\partial x_i} = \frac{\partial f(\mathbf{x})}{\partial x_i} + \sum_{j=1}^{m} \lambda_j \frac{\partial g_i}{\partial x_i} = 0 \tag{12}$$

The optimality condition can be written as

$$\sum_j e_{ij} \lambda_j = -1, \quad i = 1,2,\ldots,n \tag{13}$$

where

$$e_{ij} = \frac{\partial g_j}{\partial x_i} \bigg/ \frac{\partial f}{\partial x_i} \tag{14}$$

This equation represents the ratio of constraint to objective function gradients with respect to the design variables. These ratios can be associated with special forms of energy densities, depending on the type of constraint functions.

In this work, the optimality criteria techniques were used for solving the structural design problems with stress and displacement constraints. The computer program OPTSTAT[24] was used. It achieves a high degree of efficiency by making simple approximations for the Lagrange multipliers—the stress constraints are replaced by a single stiffness constraint ($g = \mathbf{u}^t K u - \hat{G}$) and each Lagrange multiplier is approximated as the multiplier for a single constraint.

$$\lambda_j = W/\hat{G}_j \tag{15}$$

A simple resizing formula is used to satisfy the OC given in Eq. (13). By multiplying both sides of Eq. (13) by x_i^2 and taking the square root,

$$x_i^{k+1} = x_i^k \left[\sum_{j=1}^{m} \lambda_j e_{ij} \right]^{1/2} \tag{16}$$

This recursion relation is used in place of Eq. (2). Element strain energies calculated during the analysis are used to resize the structure initially, meeting the constraints by scaling the design. If displacement constraints remain active, OPTSTAT then operates in a mode in which displacement gradients are calculated to determine the Lagrange multipliers.

Numerical Results

Structural analysis for stresses, displacements, and frequencies was done using the ANALYZE[25] finite-element program. Structural frequencies were calculated using Sturm's sequence method. Stress, displacement, and frequency constraint derivatives with respect to the decision variables were calculated using the adjoint variable (virtual load) method.

Optimization problems were solved with stress, displacement, and frequency constraints. Structural weight was the objective function, with element sizes (areas of rods and thicknesses of membrane and shear elements) as the design variables. A 10-bar truss, a 25-bar truss, a simplified fighter wing, and an intermediate complexity wing were the structures considered. All problems start with uniform values for the design variables.

Ten-Bar Truss

The classic 10-bar truss structure is shown in Fig. 1. Stress constraints of 25 ksi for each member and displacement constraints of 2.0 in. in the vertical direction at each node were imposed. The lower bound for all design variables was 0.1 in. and the loading condition was 100 kips downward applied at the lower two unconstrained nodes. The final design obtained by these algorithms agrees with those reported in the literature. CONMIN and ADS each found an optimum weight of 5061 lb in 12 analyses; NEWSUMT-A, 5079 lb in 13 analyses: IDESIGN, 5077 lb in 18 analyses; and VMCON, approaching the optimum from an infeasible design,[28] 5061 lb in 34 analyses.

For a loading condition of 50 kips upward applied at the upper two unconstrained nodes and 150 kips downward applied at the lower two nodes, Ref. 26 applies a lower limit of 22 Hz on the natural frequency in addition to the static constraints. The optimum weight obtained in this study, 4731 lb, was 61 lb lighter than the previously reported minimum weight (Table 1). When the frequency lower limit was increased to 30 Hz, NEWSUMT-A found the optimum weight as 6966 lb, an increase of 47%. For a 35-Hz frequency limit, the optimum weight was 15,258 lb, an increase of more than 200%. This problem has no nonstructural mass, thus changes in the frequency limit significantly increase the weight.

The iteration history for each optimization technique is shown in Fig. 2. Trends observed for the larger problems were apparent in these results. An infeasible starting design for NEWSUMT-A gave better results than an initial design scaled to the constraint surface; in this case, the most rapid convergence of any of the methods. The same was true for VMCON, although it did not immediately obtain a feasible design, as NEWSUMT-A did. In general, for an initial uniform feasible design, CONMIN converged the most

Table 1 Ten-bar ($\omega \geq 22$ Hz) final design

Member	Ref. 26	Present
1	24.68	24.87
2	1.08	0.10
3	24.43	25.99
4	12.87	13.12
5	0.10	0.10
6	1.96	1.97
7	13.69	13.20
8	16.47	15.34
9	17.80	17.51
10	0.10	0.10
Weight	4792	4731

Table 2 SFW design data

Material	Aluminum
Young's modulus	10 Mpsi
Density	0.1 lb/in.^{3}
Yield stress	40 ksi
Shear stress limit	28 ksi
Initial sizes	0.10 in.
Lower limit	0.01 in.
Deflection limit	3.00 in.
Load cases	2

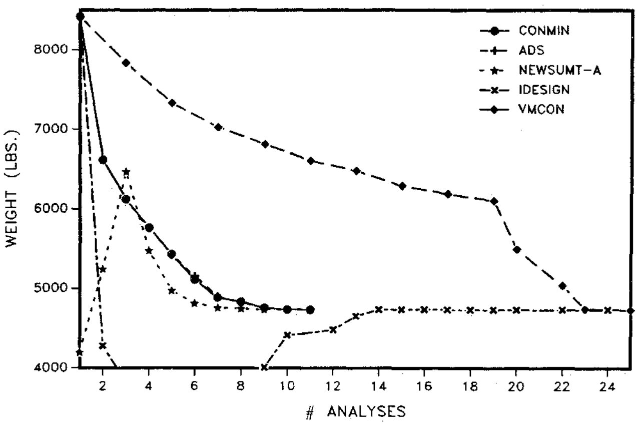

Fig. 2 Ten-bar iteration history for stress, displacement, and frequency.

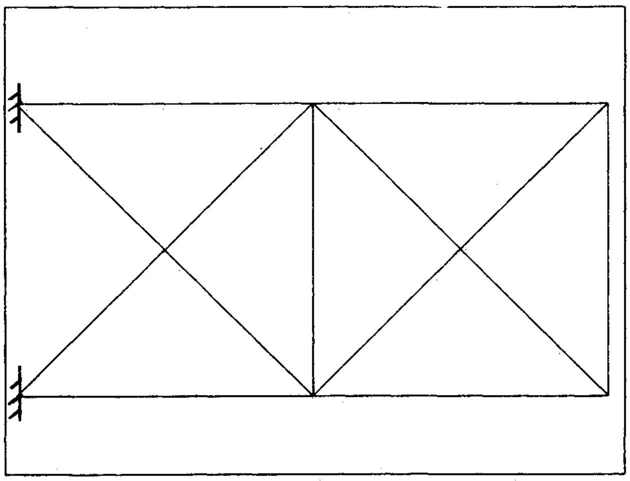

Fig. 1 Ten-bar truss model.

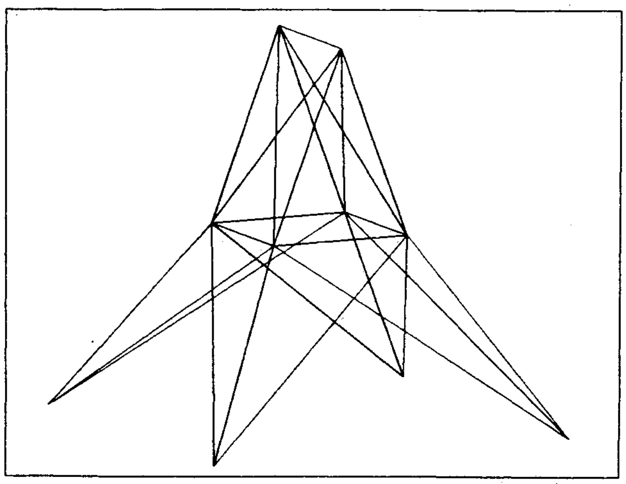

Fig. 3 Twenty-five-bar transmission tower model.

Table 3 SFW model data

Node	Geometry Coords, in.			Loads Z dir., lb		Masses, lbm
	X	Y	Z	Case 1	Case 2	
1	120	0	1.50	1500	600	0
2	120	0	−1.50	1500	600	0
3	120	12	1.50	1500	1200	0
4	120	12	−1.50	1500	1200	0
5	120	24	1.50	1500	1800	0
6	120	24	−1.50	1500	1800	0
7	100	0	2.25	0	0	2.5
8	100	0	−2.25	0	0	2.5
9	100	19	2.25	0	0	2.5
10	100	19	−2.25	0	0	2.5
11	100	38	2.25	0	0	2.5
12	100	38	−2.25	0	0	2.5
13	80	0	3.00	3000	1200	5.0
14	80	0	−3.00	3000	1200	5.0
15	80	26	3.00	3000	2400	5.0
16	80	26	−3.00	3000	2400	25.0
17	80	52	3.00	3000	3600	25.0
18	80	52	−3.00	3000	3600	25.0
19	40	0	4.50	3000	1200	5.0
20	40	0	−4.50	3000	1200	5.0
21	40	40	4.50	3000	2400	5.0
22	40	40	−4.50	3000	2400	25.0
23	40	80	4.50	3000	3600	25.0
24	40	80	−4.50	3000	3600	25.0
25	0	0	6.00	0	0	0
26	0	0	−6.00	0	0	0
27	0	54	6.00	0	0	0
28	0	54	−6.00	0	0	0
29	0	108	−6.00	0	0	0
30	0	108	−6.00	0	0	0

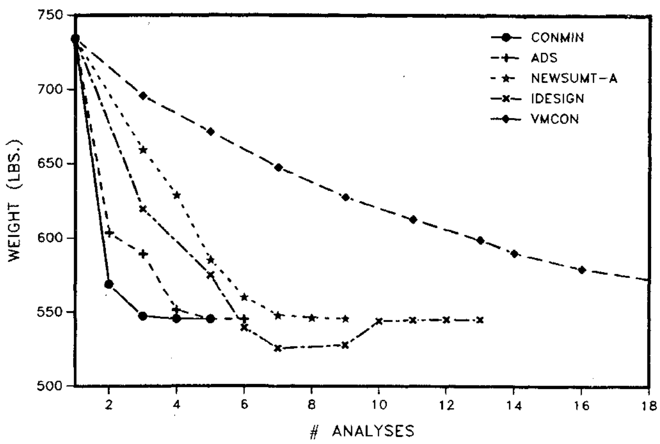

Fig. 4 Tower iteration history for static constraints.

rapidly, followed by ADS and NEWSUMT-A. IDESIGN approached the optimum from the infeasible region. VMCON, followed by IDESIGN, required the greatest number of structural analyses.

Twenty-Five-Bar Transmission Tower

The second example considered was a 25-bar space truss, which is defined in detail in Ref. 11 and shown in Fig. 3. Design variable linking was used to obtain a symmetric structure, and seven design variables were used to size the 25 members of the truss. The structure was designed for two load conditions subject to tensile and compressive stresses and displacement constraints. A comparison of the number of analyses for each technique is presented in Fig. 4. Once again, CONMIN had the most rapid convergence, VMCON the slowest, and IDESIGN approached from the infeasible region. Both RQP methods required the greatest number of analyses.

Simplified Fighter Wing

The third example was a newly formulated simplified wing structure. The model was developed to serve two purposes. First, the geometry of the wing was modeled to resemble a typical fighter delta wing: semispan of 15 ft, aspect ratio of 4, sweep angle of 24 deg, a taper ratio of 0.25, and a very thin thickness-to-chord ratio, $t/c = 0.04$. It was also intended to provide a finite-element model simple enough to check out design trends without incurring the large expense of the intermediate complexity wing used in this study. Coupling a simple finite-element model with a realistic structure resulted in conflicting requirements. The original model had 52 elements and 52 degrees of freedom. Due to the figher's knife-edge wing, the shear elements used for the spars had very high aspect ratios, which were overly stiff in the ANALYZE code. As a result, the optimization exploited the stiff spars and gave unreasonable thicknesses for those shear elements—around 5 in. thick! Furthermore, although the ANALYZE results met the displacement constraints, an analysis of the same optimum design with NASTRAN[27] gave tip deflections that were 530% larger. In fact, tip deflections for the initial design varied by 20%. As a result, the simplified fighter wing (SFW) was refined by adding another spanwise division, doubling the thickness, and shortening the semispan to 10 ft. The refined wing (Fig. 5) has 72 elements, 72 degrees of freedom, an aspect ratio of 3.67, taper ratio of 0.22, and $t/c = 0.08$. The tip displacements in ANALYZE and NASTRAN agree to within 4% for the initial design and to within 2% for the final design.

Structural weight was minimized while constraining stresses, displacements, and frequencies. The design information, displacement, and stress limits are given in Table 2, the geometry and load conditions in Table 3. In each load condition, six transverse wing tip displacement constraints and 72 stress constraints were considered, for a total of 156 behavior constraints. Design variable linking was employed and 41 design variables were used. Each top skin and spar

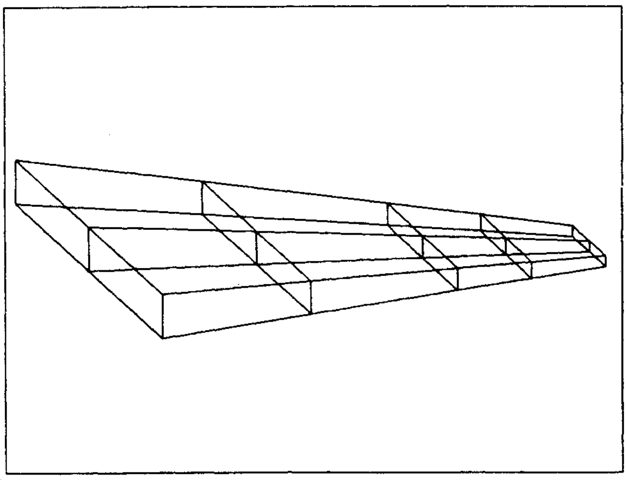

Fig. 5 SFW model.

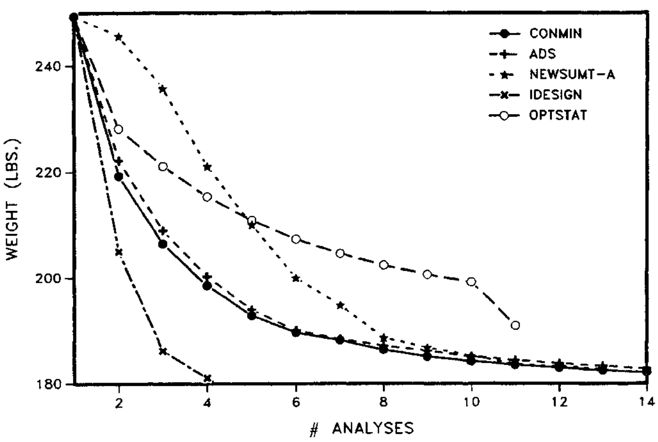

Fig. 6 SFW iteration history for static constraints.

cap was linked to its corresponding bottom skin and spar cap, and all of the posts (vertical rods) were linked into a single group. Since the loading conditions were symmetric about the midplane and the allowable stress was the same for all elements, the sizes obtained for the linked variables were the same as if they had been considered independently.

First, the SFW was designed with stress and displacement constraints. OPTSTAT performs best in the first few analyses, quickly reducing the weight (Fig. 6). The difference was more pronounced in the comparison of CPU times (Fig. 7), due to the fact that OPTSTAT does not calculate stress gradients. The lack of exact stress gradients also accounts for OPTSTAT's convergence to a design 8% heavier than the other methods. Comparisons made on the basis of CPU times were valid because all the problems were solved on the same computer (VAX 11/785) using the same analysis and gradient routines. NEWSUMT-A suffers in computational efficiency because a second-order unconstrained minimization technique incurs as high a computational cost as the analysis for a large number of design variables and constraints. The difference is less pronounced when the analysis becomes more expensive, as for frequency-constrained problems (or when constraint approximations are not used).

Displacements were the predominant constraint for the first load condition. At the optimum, two ribs had active stress constraints for the second load condition. Table 4 summarizes the optimum weights and number of finite-element analyses (FEA) required for each method. VMCON failed for this and subsequent problems; no further results are reported for it. The IDESIGN program failed to satisfy

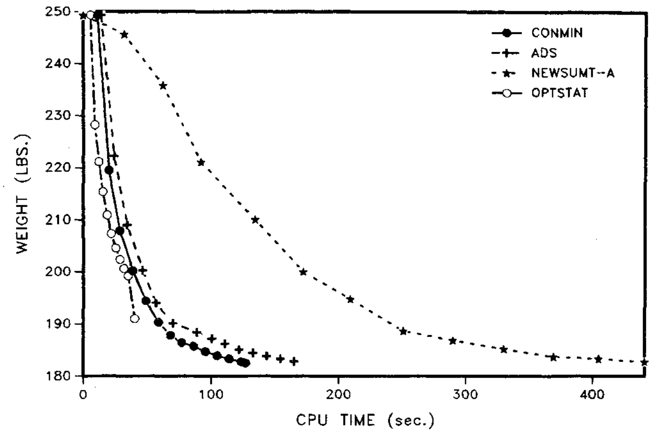

Fig. 7 SFW CPU history for static constraints.

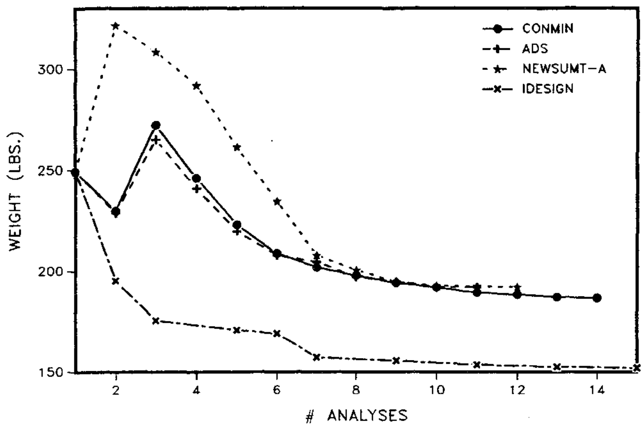

Fig. 8 SFW iteration history for static and frequency constraints.

Table 4 SFW optimization results
(stress and displacement)

Algorithm	Weight	FEA	Comment
ADS	182.8	14	
CONMIN	182.1	14	
IDESIGN	167.3	50	Infeasible
NEWSUMT-A	182.7	13	
OPTSTAT	207.4	11	
VMCON	158.2	2	Failed

constraint violations and terminate with a feasible design, even after a large number of iterations. The optimum design obtained by the other MP method differs from that reached by OPTSTAT. The former concentrates more weight in the spar caps, while the latter distributes the weight in the skins. This is because OPTSTAT does not use exact stress gradients, which contain the information that rods are more efficient load-carrying members than membranes. The slower convergence of OPTSTAT from the outset was attributable to the dominance of the displacement constraints for this case. In spite of the fact that OPTSTAT begins in the stress mode, the CPU comparison showed that a hybrid method that begins with this OC method for a few iterations would still benefit in computational savings.

Finally, a frequency constraint was added to the SFW with the nonstructural mass shown in Table 4. The fundamental frequency for the optimum stress and displacement constrained design was about 31 Hz. A lower bound of 35 Hz had no significant effect on the weight (Fig. 8). When the lower bound was increased to 40 Hz, the weight was 15%

Table 5 SFW optimization results (stress, displacement, and frequency)

Algorithm	Weight	FEA	Comment
	Frequency ≥ 135 Hz		
ADS	192.8	11	
CONMIN	188.3	12	
IDESIGN	153.1	33	Infeasible
NEWSUMT-A	192.1	12	
VMCON	—	—	Failed
	Frequency ≥ 40 Hz		
ADS	215.6	12	
CONMIN	207.1	14	
IDESIGN	154.8	36	Infeasible
NEWSUMT-A	220.7	19	
VMCON	—	—	Failed

Table 6 ICW optimization results

Algorithm	Weight	FEA	Comment
ADS	93.2	8	
CONMIN	93.3	8	
IDESIGN	88.6	36	Infeasible
NEWSUMT-A	93.2	14	
OPTSTAT	96.4	7	
VMCON	—	—	Failed

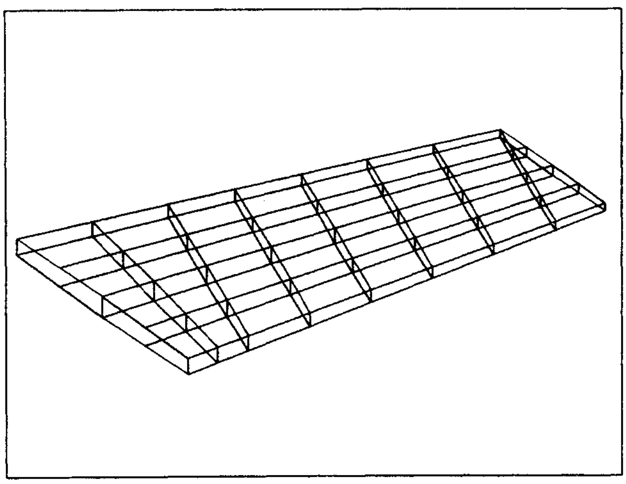

Fig. 9 ICW model.

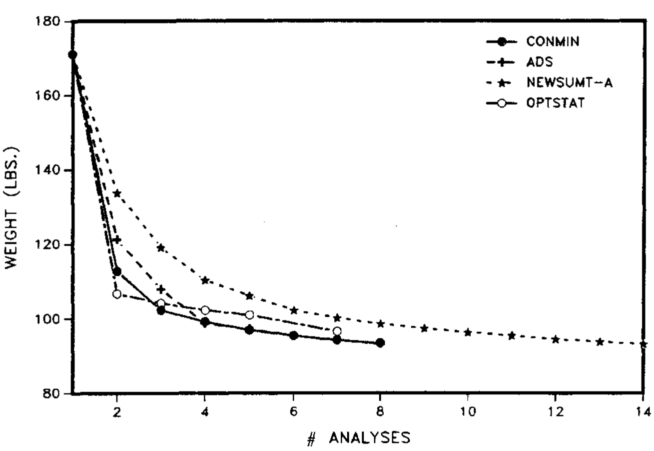

Fig. 10 ICW iteration history for static constraints.

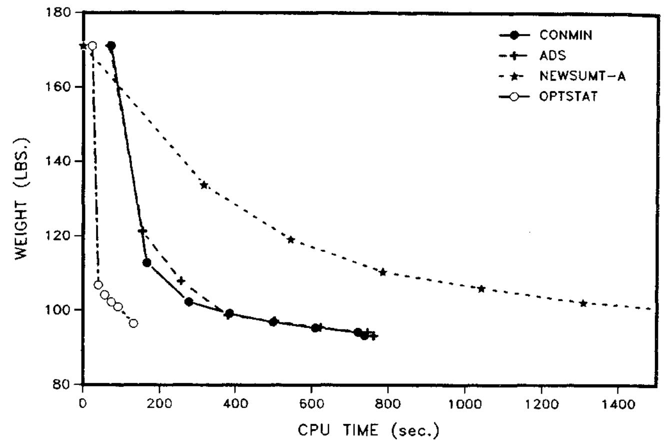

Fig. 11 ICW CPU history for static constraints.

higher. Results are given in Table 5 for all the optimization techniques employed for the SFW frequency problems.

Intermediate Complexity Wing

The final example considered was an intermediate complexity wing (ICW).[24] The structural model had 158 elements and 234 degrees of freedom (Fig. 9). Stress constraints were imposed on all of the elements (60 ksi allowable normal stress and 35 ksi allowable shear stress), and displacement constraints (10.0 in.) were imposed at the tip of the wing in the transverse direction. A total of 336 constraints were considered under two independent load conditions using 57 design variables (0.01 in. lower bound). Once again, each top skin and spar cap was linked to its corresponding bottom skin and spar cap, and all of the posts were linked into a single group. In addition, all of the ribs were linked into a single group. In this case, the optimum sizes would be nearly the same as if all elements had been considered independently, since the ribs do not make a large contribution in the optimal design. The optimum design information is given in Figs. 10 and 11 and Table 6 for all of the techniques employed.

The trends observed for the SFW hold true for the larger intermediate complexity wing. OPTSTAT converged more quickly in the first few analyses, and the computational savings were more pronounced when considering CPU times. The final designs obtained by MP methods had lower weights at a higher computational cost.

Conclusions

Optimum design of practical aerospace structures is a complicated problem because of convergence difficulties. The motivation for this study was the possible development of a hybrid optimization method using optimality criteria and mathematical programming techniques. This work was aimed at finding an efficient and reliable mathematical programming technique among the most commonly used methods. This paper compares the numerical efficiencies of different optimization techniques in solving multiple-constraint problems.

Four mathematical programming techniques were used with stress, displacement, gage, and frequency constraints. An optimality criterion method was also used with all but the frequency constraints. From the number of analyses required in each technique, the optimality technique showed better convergence during the first three to four analyses and was computationally more efficient than the MP methods. Among the MP methods, VMCON used more structural analyses than the other methods for the truss problems, and it failed to solve the wing problems. Of the remaining MP methods, IDESIGN required the most structural analyses;

however, for the wing problems, all three IDESIGN optimization algorithms (cost function, recursive quadratic programming, and a hybrid method) failed to converge to a feasible design. IDESIGN and VMCON both approached the optimum from the infeasible region, independent of the starting point. CONMIN performed as well as or better than ADS. CONMIN, ADS, and NEWSUMT-A proved to be the most reliable, with NEWSUMT-A displaying less computational efficiency. For additional details concerning the numerical results reported here, the reader is referred to Ref. 28.

As a result of demonstrated reliability, CONMIN, and possibly NEWSUMT-A, will be used in conjunction with an optimality criterion algorithm built from OPTSTAT (and a similar program for frequency-constrained problems) in order to develop an efficient multiple-constraint, hybrid optimization algorithm for large structures. Results using the hybrid method will be reported in a future paper.

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